Fourier Transform

F.T

Signal in time domain $\underset{\text{Inverse Fourier transform}}{\overset{\text{Fourier Transform}}{\rightleftarrows}}$ Signal in Frequency domain

I.F.T

Fourier Transform for aperiodic signal

aperiodic → non periodic

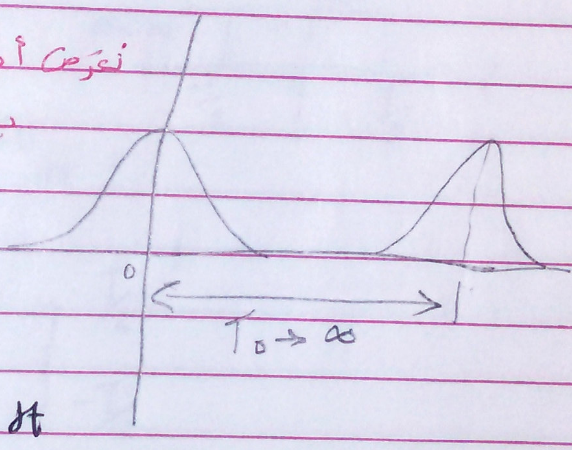
$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f_0 t}$$

نفرض أن الإشارة فترتها ∞

يعني أنها لا تتكرر

non-periodic → periodic

$$g(t) = \lim_{T_0 \to \infty} g_p(t)$$



$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p(t) \cdot e^{-j2n\pi f_0 t} \, dt$$

Assume

التغير في التردد $\Delta F = \frac{1}{T_0}$ $\qquad f_n = n f_0$

$$G(f_n) = C_n \cdot T_0$$

$$G(f_n) = \lim_{T_0 \to \infty} \int_{-T_0/2}^{T_0/2} g_p(t) \, e^{-j2\pi f_n t}$$

$$G(f_n) = \int_{-\infty}^{\infty} g_p(t) \, e^{-j2\pi f_n t}$$

$f_n \to f$

$$\boxed{G(f) = \int_{-\infty}^{\infty} g(t) \, e^{-j2\pi f t} \, dt}$$ Fourier Transform

Fourier Transform

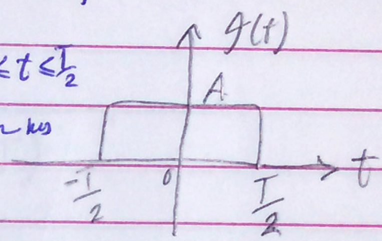
$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \, dt$$

Inverse Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \, df$$

ex:

$$g(t) = \text{rect.}\left(\frac{t}{T}\right) = \begin{cases} A & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$



$$G(f) = \int_{-T/2}^{T/2} A \cdot e^{-j2\pi f t} \, dt$$

$$G(f) = A \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-T/2}^{T/2}$$

$$G(f) = A \left[\frac{e^{-j\pi f T} - e^{j\pi f T}}{-j2\pi f} \right]$$

$$= A \left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{j2\pi f} \right]$$

$$\frac{\sin(\pi x)}{\pi x} = \text{sinc}(x)$$

$$= A \cdot T \, \frac{\sin(\pi f T)}{\pi f T}$$

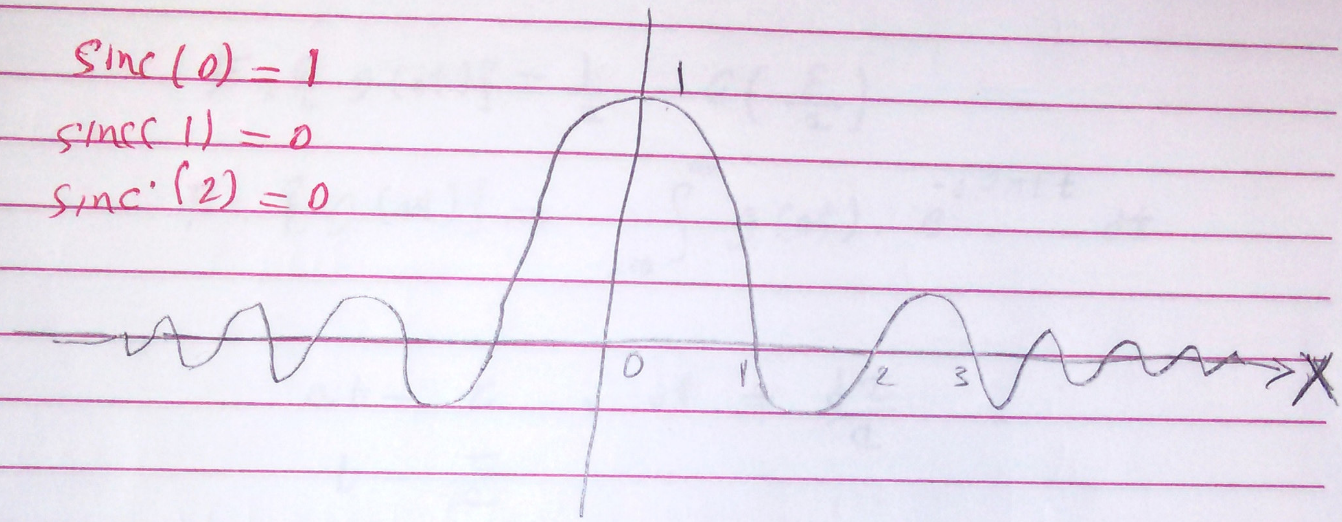
$$= A \cdot T \, \text{sinc}(fT)$$

$$\underline{\mathrm{sinc}(x)} = \frac{\sin(\pi x)}{\pi x}$$

$\mathrm{sinc}(0) = 1$

$\mathrm{sinc}(1) = 0$

$\mathrm{sinc}(2) = 0$



$$A.\mathrm{rect}\left(\frac{t}{T}\right) \longrightarrow A.T\,\mathrm{sinc}(fT)$$

$$3.\mathrm{rect}\left(\frac{t}{T}\right) \longrightarrow 3.T\,\mathrm{sinc}(fT)$$

Properties of Fourier Transform:

① Linearity

$$g_1(t) \rightarrow G_1(f)$$

$$g_2(t) \rightarrow G_2(f)$$

$$a\,g_1(t) + b\,g_2(t) \longrightarrow a\,G_1(f) + b\,G_2(f)$$

② Time Scaling

$$g(t) \rightarrow G(f)$$

$$g(at) \rightarrow \frac{1}{a}\,G\left(\frac{f}{a}\right)$$

$$2.\mathrm{rect}\left(\frac{3t}{T}\right) \Longrightarrow 2.T\,\mathrm{sinc}\left(\frac{fT}{3}\right)$$

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t}\, dt$$

$$F.\{g(at)\} = \frac{1}{a} \cdot G\left(\frac{f}{a}\right)$$

$$F.\{g(at)\} = \int_{-\infty}^{\infty} g(at) \cdot e^{-j2\pi f t}\, dt$$

$$at = \tau \rightarrow dt = \frac{d\tau}{a}$$

$$t = \frac{\tau}{a}$$

$$F.\{g(at)\} = \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f \frac{\tau}{a}}\, \frac{d\tau}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi\left(\frac{f}{a}\right)\tau}\, d\tau$$

$$F.\{g(at)\} = \frac{1}{a} \cdot G\left(\frac{f}{a}\right)$$

$$G(F) = \cancel{\tfrac{1}{a}} \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi F t}\, dt$$

(3) Time shift

$$g(t-t_0) \longrightarrow G(f) \cdot e^{-j2\pi f t_0}$$

$$\cancel{g(t)} = G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \, dt$$

$$F.\{g(t-t_0)\} = \int_{-\infty}^{\infty} g(t-t_0) \cdot e^{-j2\pi f t} \, dt$$

$t - t_0 = \tau$ $\qquad t = \tau + t_0$

$dt = d\tau$

$$F.\{g(t-t_0)\} = \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f(\tau + t_0)} \, d\tau$$

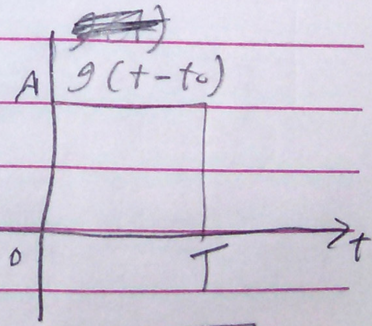
$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f(\tau)} \, d\tau \cdot e^{-j2\pi f t_0}$$

$$\therefore \; g(t-t_0) \rightarrow G(f) \cdot e^{-j2\pi f t_0}$$

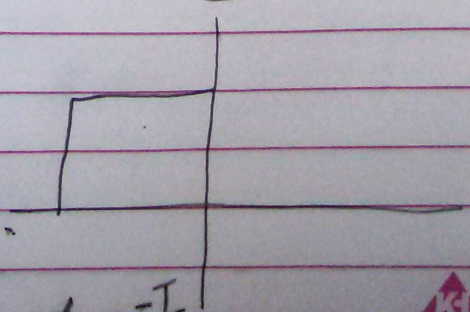
Ex

$$A \cdot rect\left(\frac{t - \frac{T}{2}}{T}\right)$$

$$\hookrightarrow A \cdot T \; sinc(fT) \cdot e^{-j\pi f T}$$



$$\boxed{t_0 = \frac{T}{2}}$$



$t_0 = \frac{-T}{2}$

④ Frequency shift

$$g(t) \rightarrow G(F)$$

$$g(t) \cdot e^{+j2\pi f_0 t} \longrightarrow G(F-f_0)$$

$$F\{g(t) \cdot e^{+j2\pi f_0 t}\} = \int_{-\infty}^{\infty} g(t)\, e^{+j2\pi f_0 t} \cdot e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi(f-f_0)t}\, dt$$

$$= G(F-f_0)$$

Sin (t) $\delta(t)$ unit Impulse

⑤ Area under $g(t)$

$$G(F) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t}\, dt$$

$$G(0) = \int_{-\infty}^{\infty} g(t)\, dt = \text{Area under Curve}$$

⑥ Area under $G(F)$

$$g(t) = \int_{-\infty}^{\infty} G(F)\, e^{j2\pi F t}\, df$$

$g(0) =$ Area under $G(F)$